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Modeling Effects of Competition on Airlines' Route-Selection Decisions

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Airlines' decisions on which routes to operate on depend on a number of factors. This paper discusses the effect of competition on such decisions. Most of the related previous work assumes that the route decisions are independently made based on the local route level characteristics, ignoring broader network properties. In our model, in addition to the effect of local route-level parameters, we include the network level significance of the route. Particularly we study how the entry or exit decisions made by two major alliances on each route are influenced by route-level parameters. Data on past decisions made by the alliances within the US air transportation system (ATS) provide inferences on such parameters. Because decision on whether or not to operate on a route is a discrete choice, our method is based on the discrete games modeling approach. The discrete games model is solved using Markov Chain Monte-Carlo (MCMC) approach to study these decisions. First, we solve for the preference parameters ignoring competition using discrete choice analysis assuming route decisions are independent. The solutions are used as priors in the MCMC technique. We introduce a novel likelihood function that considers the network level significance of routes when multiple Nash Equilibria exist. Results from our model show good agreement with actual data for the year 2014.

Nomenclature

A^1	=	player 1, Star Alliance
A^2	=	player 2, SkyTeam Alliance
r	=	route
R	=	total number of routes
s_r^k	=	strategy taken by player A^k on route r
$\vec{S}(r)$	=	(s_r^1, s_r^2): strategy profile in route r
l^k	=	change in payoff of player A^k by presence of its competitor
U_{ij}^k(r)	=	utility achieved by player A^k on route r when strategy profile $S_{ij}(r)$ is followed
V_{ij}^k(r)	=	observed component of utility U_{ij}^k
$\vec{\beta}^k$	=	preference parameter vector of player A^k
$\vec{\beta}$	=	$(\vec{\beta}^1, l^1, \vec{\beta}^2, l^2)$
$a^k(r)$	=	airport presence of player A^k in route r

I. Introduction

Airline decisions on route selection depend on several factors such as demand, operating costs, presence of competition, etc. Earlier work by the authors [1, 2] modeled such decisions assuming that the entire US air transportation system (ATS) is served by a single airline and ignored the effects of competition. This paper extends that model to include the effect of competition.

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Route selection decisions impact the topology of the air transportation network (ATS). This, in turn, affects network performance. This is why stakeholders in the ATS have an interest in knowing the airlines' decision-making model so that they can base their own decisions on them. Examples of the other stakeholders who would benefit from knowledge of the airlines' models include the FAA, which would form policies governing airline operations, organizations such as NASA, which could use these models when planning future technology development, and even competing airlines as they form their own decisions in response to the other airlines' decisions.

With this motivation, the authors' earlier work [1] developed a decision-making model for a US-based airline using demand, operating costs, geographic distances between airports, and hub / non-hub nature of airports as the inputs variables. While this work modeled demand as a continuously growing input parameter, the following work by the authors [2] developed models of passengers' decision-making. Passengers are also stakeholders in the ATS. They make decisions on which airline to fly with, and which one of the available itinerary to choose. Thus, the passengers'

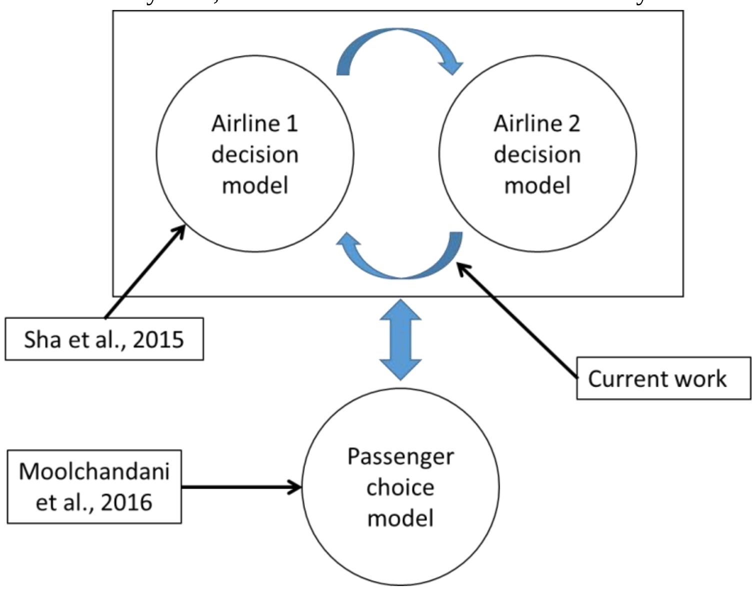

Figure 1: Schematic showing airlines' and passengers' decision models and their interaction

decision-making model was on the decisions on discrete choice between choosing a non-stop, one-stop, or two-stop itinerary. This decision model was then integrated with the airlines' route selection model in a two-level hierarchical framework. The relationship between both these models is shown schematically in Fig. 1.

Figure 1 also shows the scope of the present paper, which is to model the interaction between airlines within the ATS. Unlike previous work, this paper does not assume a single airline within the network, and instead models two competing airlines alliances based on historical data of the Star Alliance (United Airlines) and SkyTeam (Delta Airlines). The decision models of both these airlines are based on the previous work described above. The objective of the present work is to model interaction between these airlines as they account for the other's decisions within their own decision to operate or not operate on a given route.

Section II describes briefly the abstraction employed in this work. Section III gives the technical description of the method, specifically the use of Monte Carlo Markov Chains (MCMC) used for the solution of interaction game. This is followed by Section IV which presents the results, and Section V which concludes this paper.

II. Abstraction of the Air Transportation System

Among many possible abstractions of the US Air Transportation System (ATS), we choose to explore a network graph with airports as nodes and routes as links. In particular, we consider a network formed by the top 132 US domestic airports sorted based on passenger demand. There is a total of 8646 (C_2^{132}) possible routes among the shortlisted airports. We assume that a route exists in a given period for a service provider (heretofore called 'player') if the player has at least 8 scheduled departures in any two consecutive months of that period. The period is taken as a year to remove seasonal effects[5]. Routes operated by each player in a period define the topology of the network, and the topology evolves from one period to next based on the routing decisions.

Such decisions are based on the route characteristics such as passenger demand, marginal cost of adding the route, and if any (or both) of the airports that the route connects is a hub. In addition to this, the possibility of entrance on the route by a potential competitor also influences the decisions. We consider the competition between two alliances viz. Star Alliance (led by United Airlines, UA) and SkyTeam Alliance (led by Delta Airlines, DL). These alliances will be referred to as player A^1 (Star) and A^2 (SkyTeam) respectively. The words 'alliance' and 'player' will be used interchangeably in the rest of the paper.

III. Theoretical Approach

We model the route entrance decision as a game with complete information played between two alliances. The Markov Chain Monte Carlo (MCMC) technique is used to estimate the payoff functions of the airlines. The payoff functions thus derived can be used to simulate evolution of US ATS by predicting future routing decisions of the airlines using projected market data.

A. Game Theoretic Modeling

Our model of airline interaction is based on the work done in Ref. 3; out contributions are described below. On every route 'r' we model the entrance and exit decisions of each alliance as games with complete information. The strategy of each player A^k in route r is either to enter ($s^k(r) = 0$) or not enter ($s^k(r) = 1$). We assume the route decisions are independent, i.e., entry and exit decisions for a given route are not influenced by the decisions taken in other routes. Hence, the utility attained by entering a route depends only on the route characteristics. Without loss of generality, the utility of each player for not entering the route is assigned zero. Utility attained by player A^k upon entering can be written as the sum of observed (V_{ij}^k) and unobserved (ε^k) components. The observed component of utility consists of route characteristics such as market demand, route length and variation of payoff due to competition. The unobserved variables account for all the factors and effects not accounted for in this model or lack of perfect information from the perspective of the researcher. However, from the perspective of the players, this is a perfect information game.

$$U_{ij}^k(r) = V_{ij}^k(r) + \varepsilon^k(r) \tag{1}$$

For the analysis, the observed component V_{ij}^k of the utility is divided into two components: a) a component that depends on the vector of route characteristics, is represented by the vector $\vec{X}^k(r)$; b) a component that depends on the presence of the competitor. Route characteristics are route market passenger demand and route distance, which in route r The preferences for the route characteristics $\vec{X}^k(r)$ are different for different player $A^k$and is represented by preference $\vec{\beta}^k(r)$; in this paper we use demand, cost, and airport presence as the route characteristics. The presence of the competitor in the route alters this utility for player A^k by l^k. l^k would be positive or negative depending on whether the presence is beneficial or harmful. If s^{-k} is the strategy adopted by the competitor the observed component (V_{ij}^k) for player A^k can be expressed using the equation:

$$V_{ij}^k(r) = \overrightarrow{X^k}(r)\overrightarrow{\beta^k} + s^{-k}l^k \tag{2}$$

The above game can be summarized using the following payoff matrix tabulated below. In this matrix, each cell has two values of payoff, where the first value corresponds to the payoff of the first player and the second value is the payoff of the second player.

[5] This is not a weakness of the model. Seasonal effects can be analyzed by reducing the period to quarters and including that variable in the model

Table 1: Payoff matrix based on entry decision taken by each player

		(Player 2) Not Operate ($s^2 = 0$)	(Player 2) Operate($s^2 = 1$)
(Player 1)	Not Operate ($s^1 = 0$)	0,0	$0, \overrightarrow{X^2}(r)\overrightarrow{\beta^2} + \varepsilon^2$
	Operate ($s^1 = 1$)	$\overrightarrow{X^1}(r)\overrightarrow{\beta^1} + \varepsilon^1, 0$	$\overrightarrow{X^1}(r)\overrightarrow{\beta^1} + l^1 + \varepsilon^1, \overrightarrow{X^1}(r)\overrightarrow{\beta^1} + l^2 + \varepsilon^2$

B. Forward model: Nash Equilibria of the game

The game described in Sec. A is played in every route. Our aim is to estimate $\vec{\beta}$ assuming that the players play rational strategies in each route. The rational strategy for a player is to enter if the player incurs a non-negative utility. Thus, the strategy adopted by airline A^k, assuming rational behavior can be expressed as

$$s^k(r) = \begin{cases} 0 \; if \; \overrightarrow{X^k}(r)\overrightarrow{\beta^k} + s^{-k}l^k + \varepsilon^k \leq 0 \\ 1 \; if \; \overrightarrow{X^k}(r)\overrightarrow{\beta^k} + s^{-k}l^k + \varepsilon^k > 0 \end{cases} \tag{3}$$

The set of rational strategies is the Nash Equilibrium of the game. There are four pure Nash Equilibria strategies {I, II, III, IV} corresponding to strategy profile $\vec{S}(r) = \{(0,0),(1,0),(0,1),(1,1)\}$ respectively. The route characteristics, $\vec{X}^k(r)$ and the type of influence the competitor exhibits $l^k > 0 \; or \; l^k < 0$ determine whether the Nash Equilibria are non-existent, unique or non-unique. We assume that presence of competitor negatively affects the utility for both alliances, i.e., $l^1 < 0 \; and \; l^2 < 0$, the Nash Equilibria of the game in each route can be expressed as a function of unobserved variables as shown in the Fig. 2.

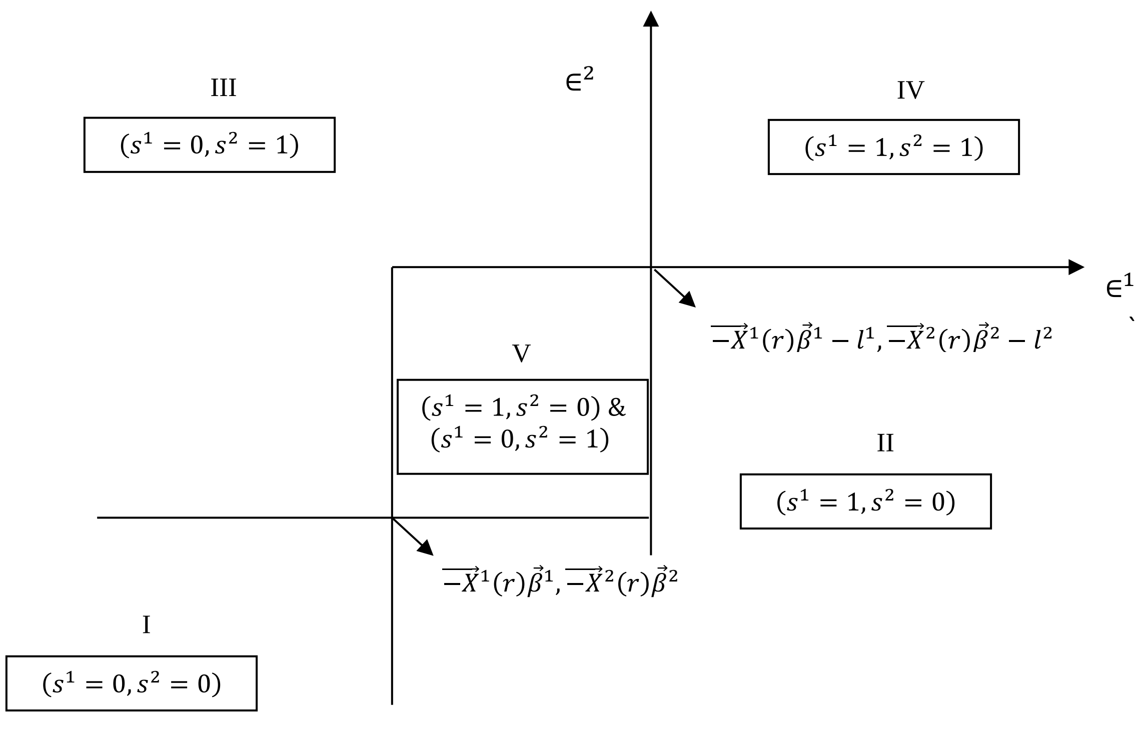

Figure 2: Nash Equilibria Regions as functions of unobserved variables assuming $l^1 < 0, l^2 < 0$

From the figure, we notice that regions I, II, III, IV have unique Nash Equilibria solutions whereas region V has multiple equilibria. Ciliberto and Tamer [12] proposed a solution that introduces a new parameter to estimate the probability of each of the pure equilibria being played conditional on the equilibrium falling in region V. However, this involves introducing large number of parameters (one for each route) leading to problems such as overfitting and poor predictive accuracy. We propose an alternate approach based on a network parameter called airport presence [12]. Airport presence of a player is the ratio of number of airports directly operated to by the player to the total number of airports directly operated to by all players from that airport. We propose that the probability of the player entering the route $p^k(r)$, conditional on the route falling in region V, is a function of average of the airport presence $a^k(r)$ of the player in the route. Our examination of historic decision indicates that the probability of a player entering is a non-linear function of airport presence which drastically changes at around 0.5. Figure 3 shows the entry decision of Player 1 as a function of its airport presence in routes where $(0,1), (1,0)$ equilibria were played for the year 2013. A logistic functional relation was assumed between the probability of entering of a player and airport presence.

$$p^k(r) = \left(1 + \alpha_1 + \alpha_2 a^k(r)\right)^{-1} \tag{5}$$

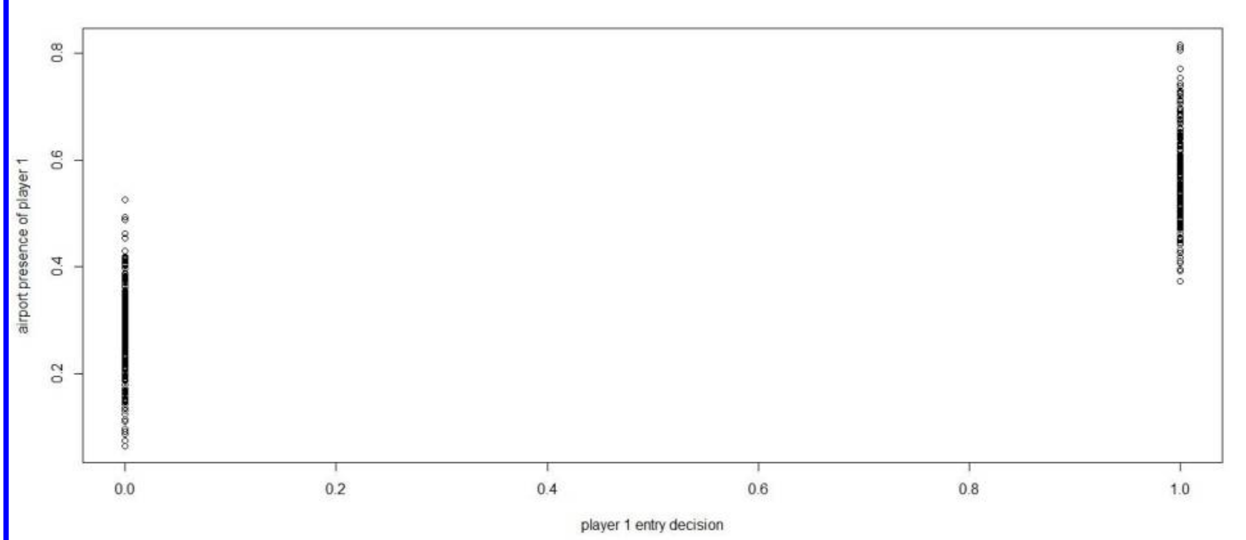

Figure 3: Entry decision taken by Star Alliance as a function of it airport presence in routes where (0,1) or (1,0) Nash Equilibria was played in 2013

C. Bayesian model for predicting the parameters

The parameters of the function in Equation 5 are obtained using the MCMC algorithm.

This subsection describes how a numerical procedure using Monte Carlo Markov Chain approach to obtain the parameters of the Game Theoretic model. The objective is to draw inferences on the parameters in game model $\vec{\beta}$

1. Prior

For each decision parameter of player k in $\vec{\beta}^k$ a normal prior is assigned, the parameters of which are obtained by running the discrete choice analysis [1]. For the parameter l^k a normal prior is assigned truncated in the region $(-\infty, 0)$, since we assume that presence of the competitor reduces the utility for both the players. α_1, α_2 is given a normal prior.

$\alpha_1 \sim N(\mu_{\alpha_1}, \sigma_{\alpha_1})$, $\alpha_2 \sim N(\mu_{\alpha_2}, \sigma_{\alpha_2})$: The parameters $\mu_{\alpha_1}, \sigma_{\alpha_1}, \mu_{\alpha_2}, \sigma_{\alpha_2}$ are obtained by fitting a logistic curve between airport presence and entry decision of player 1 in all routes where $(0,1), (1,0)$ equilibria was played. Now all routes where $(0,1), (1,0)$ equilirbia was played do not lie in region V. However, the prior is a first approximation for the coefficients and hence it's a reasonable assumption to use that for obtaining the parameters of the prior.

2. Likelihood

Assuming that the unobserved variables are independent and have a standard normal distribution, the likelihood of each of the strategy profile regions in Figure 2 can be expressed as

$$L_1(\vec{\beta}, \vec{X_r}) = \phi\left(-\vec{X_r^1}\,\vec{\beta^1}\right)\phi\left(-\vec{X_r^2}\,\vec{\beta^2}\right) \tag{6}$$

$$L_2(\vec{\beta},\vec{X_r}) = \left\{\phi\left(\vec{X_r^1}\,\vec{\beta^1}\right) - \phi\left(\vec{X_r^1}\,\vec{\beta^1} + l^1\right)\right\}\phi\left(-\vec{X_r^2}\,\vec{\beta^2}\right) + \phi\left(\vec{X_r^1}\,\vec{\beta^1} + l^1\right)\phi\left(-\vec{X_r^2}\,\vec{\beta^2} - l^2\right) \tag{7}$$

$$L_3(\vec{\beta},\vec{X_r}) = \left\{\phi\left(\vec{X_r^1}\,\vec{\beta^1}\right) - \phi\left(\vec{X_r^1}\,\vec{\beta^1} + l^1\right)\right\}\phi\left(\vec{X_r^2}\,\vec{\beta^2} + l^2\right) + \phi\left(\vec{X_r^1}\,\vec{\beta^1}\right)\phi\left(\vec{X_r^2}\,\vec{\beta^2}\right) \tag{8}$$

$$L_4(\vec{\beta},\vec{X_r}) = \phi\left(\vec{X_r^1}\,\vec{\beta^1} + l^1\right)\phi\left(\vec{X_r^2}\,\vec{\beta^2} + l^2\right) \tag{9}$$

$$L_5(\vec{\beta},\vec{X_r}) = \left\{\phi\left(\vec{X_r^1}\,\vec{\beta^1}\right) - \phi\left(\vec{X_r^1}\,\vec{\beta^1} + l^1\right)\right\}\left\{\phi\left(\vec{X_r^2}\,\vec{\beta^2}\right) - \phi\left(\vec{X_r^2}\,\vec{\beta^2} + l^2\right)\right\} \tag{10}$$

where ϕ is cumulative density of standard normal distribution. Now the likelihood function assuming that routes are independent is given by

$$L\left(\vec{X},\vec{S}\,/\,\vec{\beta},\vec{\alpha}\right) = \prod_{r=1}^{R}\left[\left\{(L_1)^{I[S_r=1]}(L_2+\lambda_r L_5)^{I[S_r=2]}(L_2+L_5-\lambda_r L_5)^{I[S_r=3]}(L_4)^{I[S_r=4]}\right\}\left\{\frac{1}{1+e^{\alpha_1+\alpha_2 a_r^1}}\right\}^{I[S_r=2\ or\ 3]}\right]$$

Where λ_r is a Bernoulli variable that assumes 1 when (1,0) equilibria played and 0 when (0,1) plays (11)

3. Posterior

Now a joint posterior density function for the parameters $\vec{\alpha},\vec{\beta}$ can be obtained applying Bayes Rule as

$$Y\left(\vec{\beta},\vec{\alpha}/\vec{X},\vec{S}\right) \propto \pi(\vec{\alpha})\pi(\vec{\beta})L\left(\vec{X},\vec{S}/\vec{\beta},\vec{\alpha}\right)$$

$$Y\left(\vec{\beta},\vec{\alpha}/\vec{X},\vec{S}\right) \propto \left(e^{\frac{-\vec{\alpha}^T\vec{\alpha}}{2}}\right)\left(e^{\frac{-\vec{\beta^1}^T\vec{\beta^1}-\vec{\beta^2}^T\vec{\beta^2}}{2}}\right)\pi(l^1)\pi(l^2)L\left(\vec{X},\vec{S}/\vec{\beta},\vec{\alpha}\right) \tag{12}$$

D. MCMC algorithm

The following steps are followed to estimate the posterior distribution.

1. Initialize $\vec{\alpha},\vec{\beta}$ using the prior

2. For each route, obtain $\lambda_r \sim Bernouli(p_r)$, where $p_r = (1+\alpha_1+\alpha_2 a_r^1)^{-1}$

3. Metropolis-Hastings (MH) algorithm to update the parameters $\vec{\theta} = (\vec{\alpha},\vec{\beta})$, $\vec{\lambda} = \{\lambda_1,\lambda_2,\dots,\lambda_R\}$. In the MH algorithm the using the prior distribution as jump function; new sample is accepted as,

$$\begin{cases} if\ c \geq 1, \vec{\theta}_{new}\ is\ accepted \\ if\ c < 1, \vec{\theta}_{new}\ is\ accepted\ with\ probability\ c \end{cases}, \text{ where } c = \frac{Y\left(\vec{\theta}_{new},\vec{\lambda}\middle|\vec{S}\right)}{Y\left(\vec{\theta}_{old},\vec{\lambda},\middle|\vec{S}\right)} \times \frac{J\left(\vec{\theta}_{old}\middle|\vec{\theta}_{new}\right)}{J\left(\vec{\theta}_{new}\middle|\vec{\theta}_{old}\right)}$$

IV. Results

In our previous work [1], we ran the model of airlines' decision-making without accounting for competition between alliances. The mean preference parameters for cost, demand and distance were obtained as -7.008 and 0.133 and -0.067 respectively. The negative sign indicates that as distance between routes increases the probability of the route getting added decreases, whereas it is positive proportional to increase in demand. These values serve as parameters for prior distribution for each of distance, demand and cost.

The model was run for a network consisting of 74 routes in the year 2013. Upon implementing the MH algorithm for the network posterior distribution on each of the parameters were obtained as shown in Figure 4.

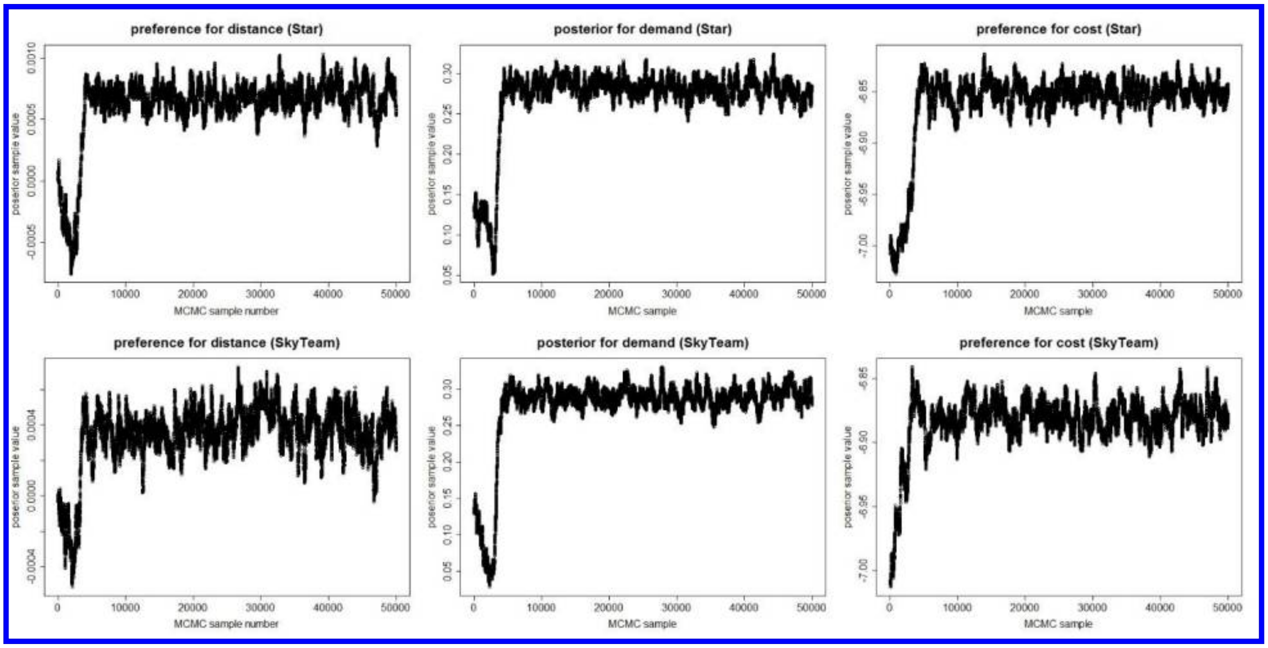

Figure 4: Posterior samples of the preference parameters for cost, demand and distance

Figure 5 shows the posterior samples of demand, distance and cost preference parameter for each of the players after providing a burn-in period of 20000

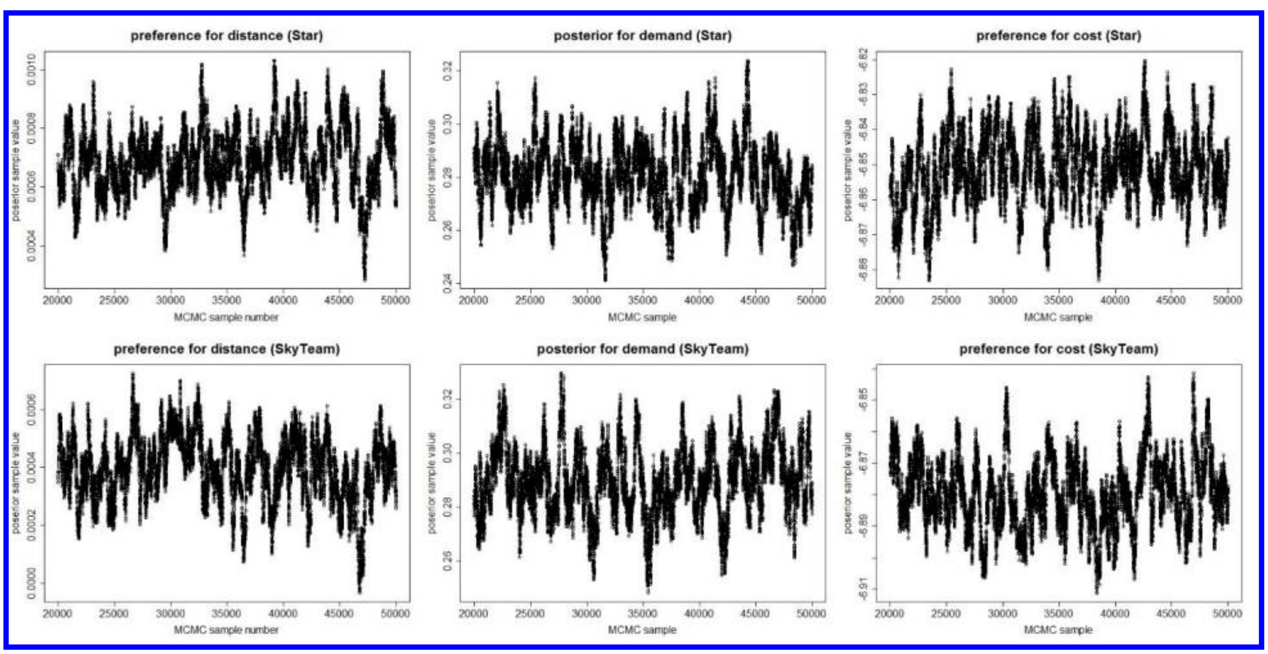

Figure 5: Posterior samples for the preference parameters after burn-in period

The results for the interaction factor (which shows the effect of the entrance of the competitor on the utility for each players) are shown in Figure 6.

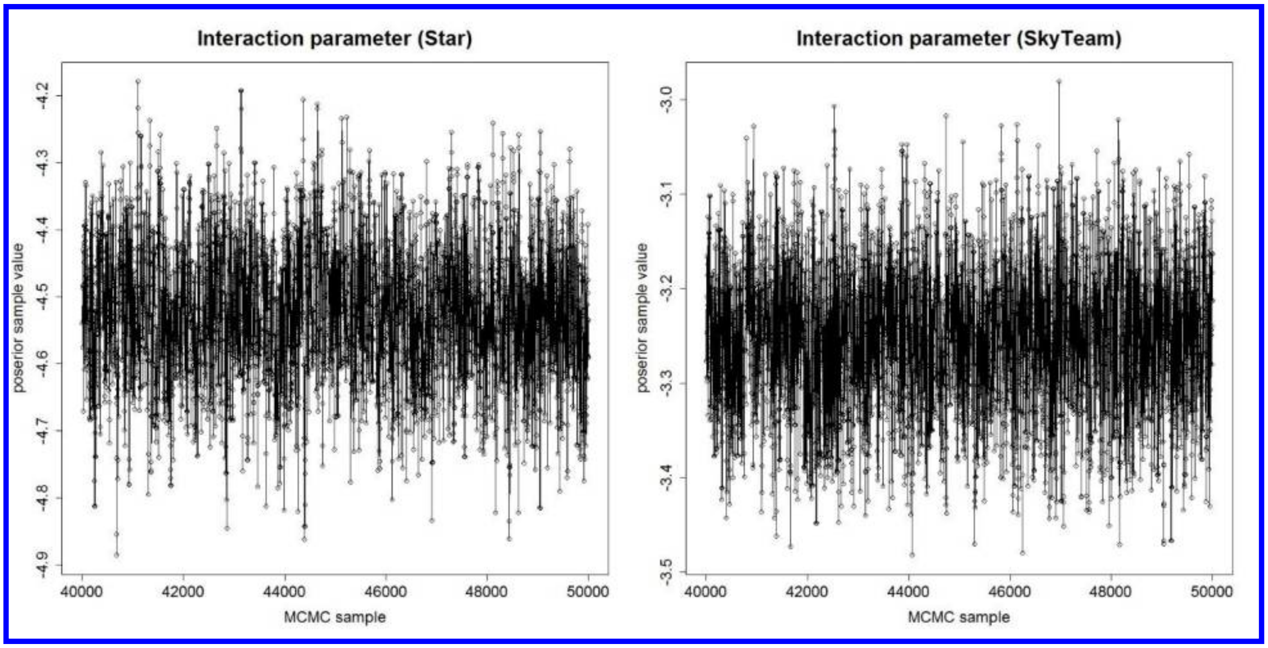

Figure 6: Posterior samples for the interaction parameters of each player

The final decision model for both the players after implementing the game theoretic model is described by the values of the preference parameters. This is summarized in Table 2. Preference for distance parameter was negligible while taking the decision (as observed from its low magnitude of posterior). Cost preference parameter had a negative magnitude while demand had positive magnitude, indicating that higher the demand and lower the cost more preferred the route is to operate. Cost preference parameter had a much higher magnitude than all other decision parameter indicating cost is the most significant decision variable of the three. Further Star had a larger decline in payoff from the presence of the competitor as compared to SkyTeam.

Table 2: Statistics of the posterior estimates on decision parameters after burn-in period

Parameter	Prior mean	Posterior mean	Posterior standard dev
Cost(Star) β_1^1	-7.008	-6.851	0.012
Demand(Star) β_2^1	0.133	0.281	0.010
Cost(SkyTeam) β_1^2	-7.008	-6.880	0.013
Demand(SkyTeam) β_2^2	0.133	0.300	0.011
Interaction(Star) l^1	-100	-4.530	0.010
Interaction(SkyTeam) l^2	-100	-3.251	0.077
α_1	-21	-29.910	0.495
α_2	49	42.322	0.669

Using the mean posterior estimates for coefficient on airport presence, conditional on region V – the probability of Star Alliance entering as predicted by the model, a function of its airport presence is plotted in Figure 7.

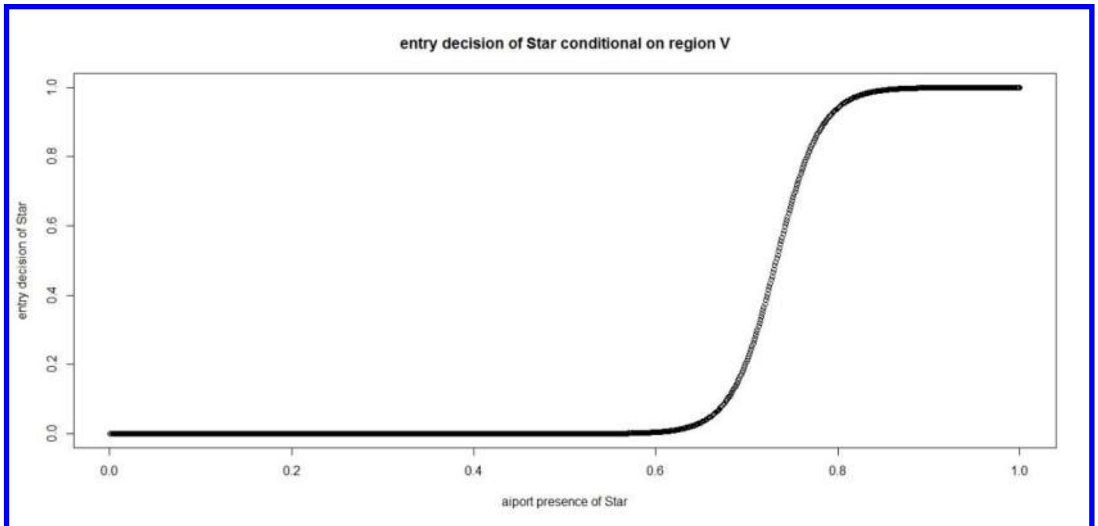

Figure 7: Entry decision of Star Alliance as a function of airport presence conditional on Region V

Now based on the estimates of preference parameters the routes can be partitioned based on the Nash Equilibrium zone it falls. The partitioning as predicted by the model is compared with actual Nash Equilibrium in the Table 3.

Table 3: Tabulating the partitioning of the routes based on the Nah Equilibria played and Region it falls in

Nash Equilibrium	No. of routes (actual)	Region	No. of routes (model)
(0,0)	0	I	1
(1,0)	61	II	48
(0,1)	2	III	3
(1,1)	11	IV	5
		V	17

Using the information the model was used to predict the Nash Equilibrium to which the route falls in based on route characteristics such as demand, cost, and airport presence. The prediction of the model is compared to the data as shown in the bar chart (Figure 8).

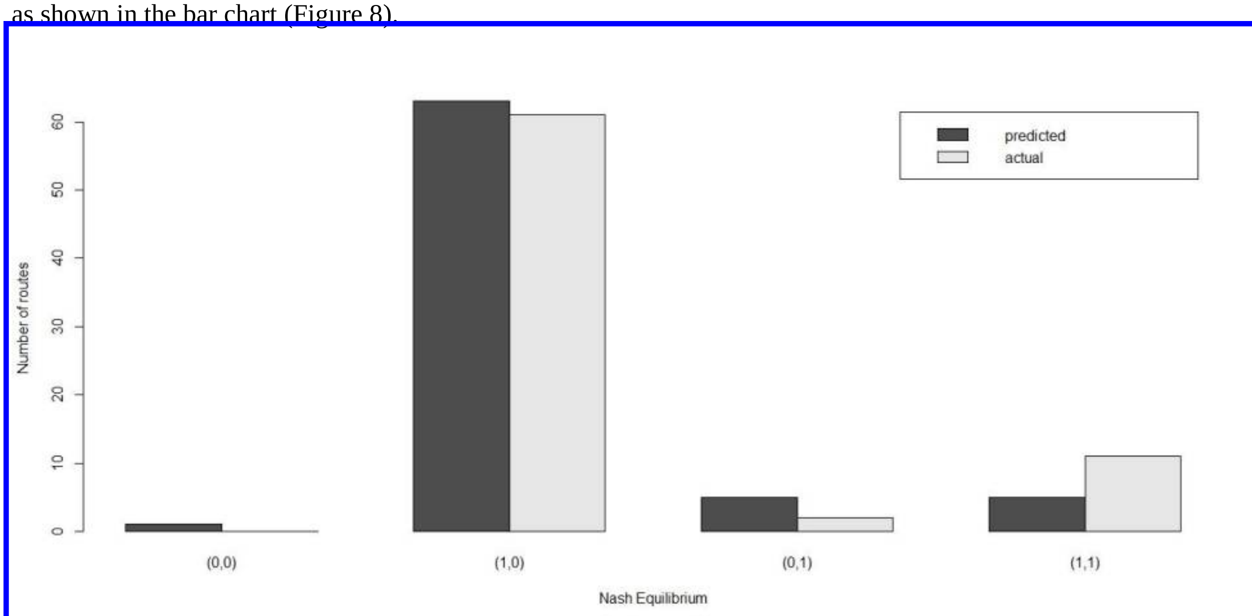

Figure 8: Verification: Predicted Nash Equilibria against actual Nash Equilibria strategy

From figure 8, we observe good comparison of model classification of equilibria with observed equilibria for the same year. Thus the proposed model can capture the decision model of the airlines using much fewer number of parameters. This gives advantages such as better prediction accuracy and no overfitting.

Following this, our future plans include testing for different implementations of the MH algorithm using various priors, jump function and in addition to integrate this model within the hierarchical decision-model of airlines and passengers.

V. Conclusion and Ongoing Work

Based on the game theoretic model developed preference parameters of each player (airlines) for each of the decision parameter such as distance, cost and demand was obtained. It was observed that cost was the most important decision variable for both the players. The model also showed that Star Alliance had a higher disutility in the presence of its competitor as compared to SkyTeam.

The present results are shown for an assumed prior. The process will be repeated for different priors to study if improvement in accuracy is achieved by changing priors. The accuracy so obtained will be compared with the earlier ones achieved by discrete choice analysis ignoring competition. Interaction of airline competition factor with other decision variables such as market demand, cost etc. will be studied. Overall, the work will significantly contribute to gaining better insights on airline decision prediction and strategic analysis from limited available data. This work will also be integrated into the hierarchical decision-making model shown in Fig. 1. Once two airline alliances with separate decision models are developed, the passengers will then have to make decision on which one to fly with, and this would be developed to be integrated in the hierarchical decision model.

Among the three decision variables viz. distance, demand and cost – only cost is under the control of policy makers such as FAA. It is encouraging to note from the results that cost is the most significant among the three in the decision making process of the airlines. Hence the inferences gained from these model can be used by such federal agencies to study the impact of various policies or US ATS topology and can guide the policy makers to frame policies. In future, such policy experiments will be conducted to provide valuable recommendations for FAA.

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